10p

N64 12401*
code 1

~~Report~~ NAS-585-8

15 June 1963

(2) Memorandum Report (NASA CR-55101;

OTS: $1.10ph, $0.80mf

on

THE THEORY OF THE CONDUCTIVITY PROBE

by

W. W. Zachary

15 Jun. 1963 10p refs

OTS PRICE

XEROX $ 1.10 ph

MICROFILM $ 0.80 mf.

(2)

Submitted to

National Aeronautics and Space Administration

under

(NASA Contract NAS 5-585)

by

2920254

ELECTROMAGNETIC RESEARCH CORPORATION

~~5001 COLLEGE AVENUE~~

COLLEGE PARK, MD.

RQT-11619

ELECTROMAGNETIC RESEARCH CORPORATION

THE THEORY OF THE CONDUCTIVITY PROBE

by

W. W. Zachary

I. INTRODUCTION

The object of this memorandum is to discuss some preliminary results which have been obtained in an attempt to improve upon the theory of the conductivity probe. The existing theory of this device is given by Smith [1]*, and is summarized below for purposes of completeness.

Consider two metallic spheres of unequal surface area which are electrically connected, but physically isolated. As is indicated in Fig. 1, a current flows

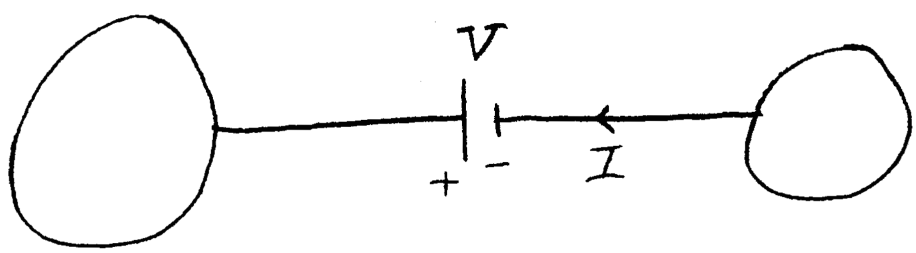

Fig. 1

as a result of the applied voltage. The quantities V and I are the experimentally measured parameters. The theory given by Smith relates these quantities to the capacitances of the spheres, and the polar conductivites, $\sigma+$ and $\sigma-$, of the plasma in which the apparatus is immersed. That is, $\sigma+$ is the conductivity of the positively charged particles and $\sigma-$ is the conductivity of the negatively charged particles of the plasma. The voltage polarity shown in Fig. 1 is for purposes of illustration. In practice, the polarity is changed

* Numbers in brackets refer to the corresponding numbers in the References on p. 9.

during the experiment [1].

In Section II we discuss the validity of the assumptions involved in Smith's theory and indicate those which should be either discarded or improved upon. In Section III we give a derivation of the positive ion conductivity which should be substituted for the expression used by Smith in his data analysis (the restricted validity of which is discussed in Section III). Finally, Section IV is devoted to some concluding remarks.

II. SOME REMARKS ON SMITH'S ANALYSIS

In order to understand the philosophy behind the present approach, it is necessary to examine the assumptions made in Smith's analysis. We first list them without comment.

(a) space charge is neglected

(b) the terrestrial magnetic field is neglected

(c) the plasma is assumed homogeneous

(d) the radius of curvatures of the sheaths formed on the surfaces of the spheres are small compared to the radius of the smaller sphere (This assumption is not independent of assumption (a).)

(e) the charged particle densities are so small compared to the neutral particle density that collective effects may be ignored

(f) all effects of the vehicle motion on the probe performance are neglected

Now, in fact, Smith observes a linear current-voltage characteristic which indicates that assumptions (a) and (d) are legitimate. A check on the tabulated data for the D-region [2] shows that assumption (e) is satisfied. If (a) and (d) are satisfied, then it is reasonable to expect that (c) will also be satisfied. Qualitatively speaking, one would not expect the magnetic field to have a large effect on the magnitude of the conductivites, so that we tentatively suppose that assumption (b) is legitimate. This leaves only assumption (f) in doubt. We feel that this should be further investigated. In particular, if the vehicle velocity is greater than the thermal velocity of any of the plasma components, then the particle distribution near the vehicle surface can be greatly affected. In turn, the probe performance can be expected to change. This point will not be investigated further here.

It is not so much Smith's analysis that we question here, but rather his interpretation of the data. To illustrate this point, we consider the determination of the positive ion density from the measured values of the voltage and current.

From the measured values of current and voltage, Smith determines values for the positive ion conductivity. In order to proceed further, one needs to know the functional dependence of the conductivity on the ion density. Smith takes this to be,

$$\sigma_+ = \frac{n_+ e^2}{m_+ \nu} \tag{1}$$

where $m+$ = ion mass

$\sigma+$ = positive ion conductivity

$n+$ = positive ion density

e = electronic charge

ν = collision frequency of positive ions with the neutral particles

It is the justification of this step that we question.

Equation (1) is valid for charged particles moving freely among themselves, but colliding with particles of another species. However, the assumption usually made in the derivation of (1) is that the mass of the charged particle is much less than that of the particles of the second species. In the present case, however, we are interested in the situation in which the two types of particles are of comparable mass. Of course, it is possible that (1) is correct. Whether it is or not depends upon the definition of the collision frequency, ν. Smith is somewhat nebulous concerning the source of his data for this quantity. For these reasons, in the remainder of this report we consider a simple model to see if (1) is correct.

III. DERIVATION OF POSITIVE ION CONDUCTIVITY

We seek below an expression for the conductivity of ions scattering against neutral particles with no restriction on the relative sizes of the masses of the two particles.

There has been much work done on this problem. We refer to the books by Loeb [3] and Massey and Burhop [4] for reviews of the subject. An approach from a more mathematical viewpoint is that of Wannier [5], who gives a result which is directly applicable to the present problem.

It turns out that the case in which the two types of particles have comparable masses is difficult to work out. Indeed, part of the calculation requires numerical methods. Fortunately, this has already been performed by Wannier.

In order to calculate the conductivity, some assumption must be made concerning the force law acting between the particles. Since, in any event, we are dealing with a short range force, the result should not depend critically upon this choice. In the light of these remarks it is convenient to assume a Maxwell force. This means that the scattering potential has the form,

$$\varphi(r) \sim r^{-4}$$

where r is the interparticle distance. Actually, it is known that this is the correct force at large interparticle distances [4]. The advantage in choosing this particular force law is that the relative velocity then cancels out of the collision integral. The presence of this quantity complicates the treatment of general force laws.

With this assumption, Wannier [5] finds for the average velocity in the field direction $\langle v_z \rangle$,

$$\langle V_z \rangle = \frac{.7048}{2\pi} \frac{E}{N P^{1/2}} \left(\frac{1}{M} + \frac{1}{m_+} \right)^{1/2} \tag{2}$$

where M = mass of neutral particles

E = external electric field corresponding to the applied voltage

N = neutral particle density

P = polarizability of the neutral particles

The polarizability is given by [4],

$$P = \frac{K - 1}{4\pi N_L} \tag{3}$$

where K = dielectric constant of the medium

N_L = Loschmidt's number = 2.7×10^{19} cm^{-3}

The positive ion conductivity is then,

$$\sigma_+ = \frac{.7048}{2\pi} \frac{n_+ e}{N P^{1/2}} \left(\frac{1}{M} + \frac{1}{m_+} \right)^{1/2} \tag{4}$$

Assuming that $M = m_+ \cong 30$ amu, using Smith's measured values of n_+ and using tabulated values of N [2] we find that the values of σ_+ from (4) differ from Smith's measured values by a factor of approximately 2.5.

It is instructive at this point to examine more closely the physical ideas contained in (4). It is well-known that it is more advantageous, when discussing particles interacting by means of the polarization force, to define a mean free time between collisions instead of a mean free path, since the collision cross section varies inversely as the approach speed of the particles.

The mean free time is defined by [5],

$$\tau = \frac{1}{N \Sigma \gamma} \tag{5}$$

where N = neutral particle density

Σ = total collision cross section

γ = relative velocity of two particles undergoing a binary collision

Thus τ is independent of γ, since Σ is inversely proportional to γ.

The mean velocity of the charged particle in the electric field direction is given by [5],

$$\langle V_z \rangle = eE\left(\frac{M+m}{m^2}\right)\frac{1}{\left\langle \frac{1-\cos\chi}{\tau} \right\rangle} \tag{6}$$

The result given in (2) is contained in (6), although, as will be seen below, the evaluation of the average over the collision process of $\frac{1-\cos\chi}{\tau}$ is quite involved.

According to classical collision theory the scattering angle, χ, is given by:

$$\chi = \pi - 2\int_0^{u_1} \frac{du}{\left[\frac{1}{b^2} - u^2 + \frac{e^2 P (M+m)}{M m b^2 \gamma^2} u^4\right]^{1/2}} \tag{7}$$

where b is the so-called impact parameter and u_1 is the lower of the two positive roots of the polynomial in the denominator. If the polynomial has no real roots, then the upper limit of the integration is infinity.

The existence of a real root depends upon the nature of the orbit. If b is sufficiently large, a real root exists and the orbit is one branch of a hyperbola. For sufficiently small b, no root exists and the particles approach each other in spiralling orbits. These two classifications of orbits are separated by a limiting orbit in which the particles spiral asymptotically into a circular orbit. This limiting orbit is found by setting the polynomial in (7) equal to zero. Then one finds,

$$b_\ell = \left[\frac{4e^2 \rho}{\gamma^2} \left(\frac{1}{M} + \frac{1}{m_+} \right) \right]^{1/4}$$

and in addition,

$$\tau_s = \frac{1}{2\pi e N} \left[\left(\frac{1}{M} + \frac{1}{m_+} \right) \rho \right]^{-\frac{1}{2}} \tag{8}$$

the subscript s denoting spiralling collisions.

The calculation of τ for hyperbolic orbits is more difficult and, in general, requires numerical methods. The above analysis shows, however, the origin of the square root factors in (2), which do not occur in (1).

Proceeding from (6), Wannier derives (2) taking into account both the hyperbolic and spiral orbits. The fact that τ_s is constant (independent of relative velocity) produces significant simplification.

In concluding this section we note that one need not be concerned with such involved orbit considerations to derive (1). However, (1) does not involve sufficient physical consideration for the present problem.

IV. CONCLUDING REMARKS

In view of the above discussion we conclude that (1) is not adequate for the purposes of the present problem. In general, we may say that when particles of comparable mass are colliding, one must take careful account of the types of particle orbits which occur. (1) does not contain enough detail for this purpose.

The conclusion to be drawn from the above discussion is that Smith's (implicit) assumption that the mass of the positive ions is much less than the mass of the neutral particles is not valid.

The correction to Smith's analysis for ion mass is not, however, sufficient to give agreement with experiment. As mentioned previously, we propose that the corrections to the theory due to effects of the vehicle velocity will close the gap. These effects should represent the major contribution in view of the argument given above that effects due to the terrestrial magnetic field and space charge should not be very important.

REFERENCES

[1] L. G. Smith, Progress Report No. 3 on Contract NASW-98, Geophysics Corp. of America, Boston, May, 1960.

[2] U. S. Standard Atmosphere, 1962.

[3] L. B. Loeb, Basic Processes of Gaseous Electronics, Univ. Calif. Press, 1960, Second Edition, revised.

[4] H. S. W. Massey and E. H. S. Burhop, Electronic and Ionic Impact Phenomena, Oxford, 1952.

[5] G. H. Wannier, Bell Syst. Tech. J. **32**, 170 (1953).